of availability of many first-order eigenvalue subroutines with the disadvantage of more required computer storage.

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Marangoni Effect and Capacity Degradation in Axially Grooved Heat Pipes

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Introduction

THE axially grooved heat pipe employs capillary grooves in the pipe wall to return condensed fluid to the evaporator. Because of its simplicity and low cost, the axially grooved heat pipe has received considerable attention. It was successfully employed as a key thermal-control element on the ATS-6 satellite and has been selected for the Space Shuttle thermal canister to provide a closely controlled thermal environment for astronomical instruments. The latter application requires several gas-controlled variable-conductance heat pipes. Numerous investigators have observed, however, that when used with a control gas the axially grooved heat pipe suffers a marked capacity degradation. This degradation has been especially perplexing when one considers that conventional theory predicts a capacity increase as the gas blockage in the condenser shortens the active length.

Until now, no proposed mechanism for the observed degradation has been sufficient to explain it. We believe that the degradation primarily results from the Marangoni effect, which is liquid set in motion by a surface-tension gradient. In a gas-loaded variable-conductance heat pipe, a large surface-tension gradient can occur because of the temperature gradient in the gas-blocked zone. With ammonia, for example, the surface tension is 2.6 times higher just above its melting point than at 50°C. The surface-tension gradient produces a shear on the surface of the liquid in the gas-blocked region, and hence a surface flow is induced toward the condenser end. This is accompanied by a subsurface flow in the groove of equal magnitude but opposite direction. The pressure drop that drives the subsurface flow subtracts directly from the capillary pressure available to return the condensate to the evaporator. Figure 1 shows this situation graphically, where the liquid pressure variation at the capillary-pressure limit with Marangoni flow is compared to the variation with conventional theory.

Procedure

To model the Marangoni effect, an integral rather than a differential appraoch is taken, which more clearly shows how various forces act on the liquid. Consider as a free body the liquid in a single groove (Fig. 2). The groove is rectangular in cross section, with depth t, width w, length L and inclination α. The active region is of length L_a and the gas-blocked region is of length L_g. The imaginary surfaces that form the boundary of the free body are taken just on the liquid side of the groove wall and just on the vapor side of the meniscus. In steady state, the forces on the liquid must sum to zero. In calculating the forces we assume that the pressure P_v of the vapor and gas above the groove is uniform and that there is no shear on the liquid surface from the vapor flow. The force components parallel to the groove acting in the direction toward the evaporator are:

$P_\ell A|_0$ = liquid pressure x cross-sectional area at the condenser end

$\sigma s|_L$ = surface tension x meniscus perimeter at the evaporator end

and those components in the direction toward the condenser are:

$P_\ell A|_L$ = liquid pressure x cross-sectional area at the evaporator end

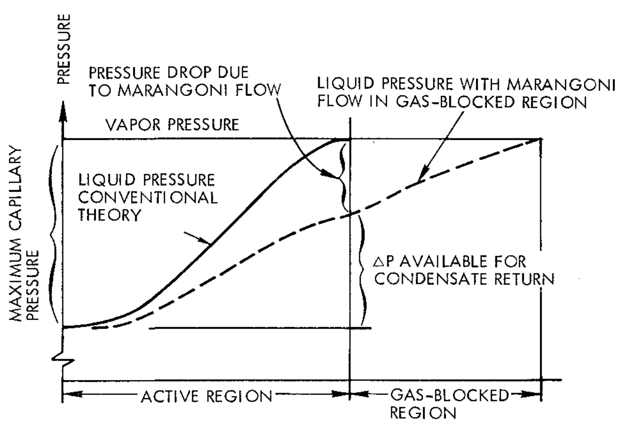

Fig. 1 Pressure variations along a groove for Marangoni and conventional theories.

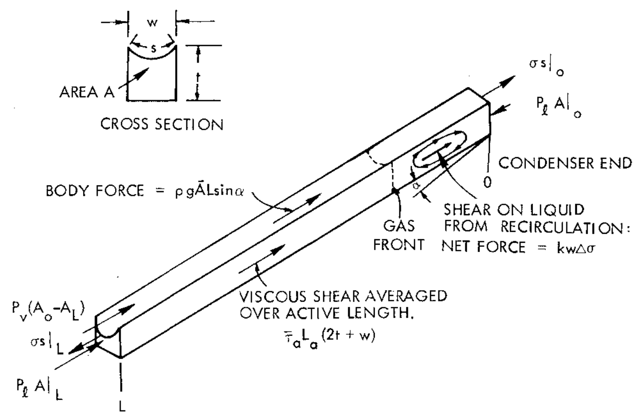

Fig. 2 Free-body diagram of liquid in groove with parallel force components.

Presented as Paper 78-458 at the 3rd International Heat Pipe Conference, Palo Alto, Calif., May 22-24, 1978; submitted June 27, 1978; revision received March 3, 1979.

Index category: Heat Pipes.

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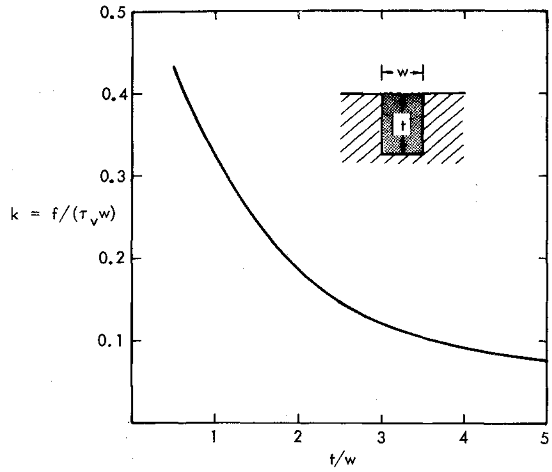

Fig. 3 Ratio of net shear on groove wall to free-surface shear.

$P_v(A_0 - A_L)$ = vapor and gas pressure x projected area of meniscus

$\rho g\bar{A}L\sin\alpha$ = component of liquid weight parallel to the groove ($\bar{A}$ is the average cross-sectional area)

$\bar{\tau}_a L_a(2t+w)$ = viscous shear averaged over the active length x wetted area

$\sigma s|_0$ = surface tension x meniscus perimeter at the condenser end,

$kw\Delta\sigma$ = net viscous force due to recirculation in the gas-blocked region

We will derive presently this last component. For now we use the result that it is proportional to the product of the groove width w and the surface-tension difference $\Delta\sigma = \sigma_0 - \sigma_L$, with the proportionality constant k a function of the groove geometry as given in Fig. 3.

The force components are summed, and the sum is set to zero. In doing so we assume that the liquid contact angle is 0 deg and the groove is operating at the capillary limit so that the meniscus at the evaporator end is circular and fully recessed. The cross-sectional area, meniscus perimeter, and liquid pressure are given by

$$A_L = wt - \pi w^2/8, \quad s_L = \pi w/2, \quad P_\ell|_L = P_v - 2\sigma_L/w$$

We also assume that there is sufficient fluid that the meniscus at the condenser end is flat, and therefore

$$A_0 = wt, \quad s_0 = w, \quad P_\ell|_0 = P_v$$

For the average cross-sectional area $\bar{A}$ we take

$$\bar{A} = \tfrac{1}{2}(A_L + A_0) = wt - \pi w^2/16 \tag{1}$$

The sum of the force components set equal to zero can now be written as

$$\frac{2\sigma_L}{w}\left(wt - \frac{\pi w^2}{8}\right) + \sigma_L\left(\frac{\pi w}{2} - w\right) - (1+k)\Delta\sigma w$$
$$-\rho g\left(wt - \frac{\pi w^2}{16}\right)L\sin\alpha - \bar{\tau}_a L_a(2t+w) = 0 \tag{2}$$

The watt-meter capacity of the heat pipe $\dot{Q}L_{\rm eff}$ is proportional to $\bar{\tau}_a L_a$. To show this, consider that $\dot{Q}L_{\rm eff}$ is given in terms of the local mass flow $\dot{m}(z)$ in the active region and the latent heat λ by

$$\dot{Q}L_{\rm eff} = \lambda\int_0^{L_a}\dot{m}(z)\,dz \tag{3}$$

where the gas front is at $z=0$. Since the mass flow is proportional to the local average shear τ_a, then

$$\dot{Q}L_{\rm eff} \propto \int_0^{L_a}\tau_a\,dz = L_a\bar{\tau}_a \tag{4}$$

To construct an expression for the ratio of the degraded watt-meter capacity $\dot{Q}L_{\rm eff}$ to the capacity of the isothermal heat pipe without degradation $(\dot{Q}L_{\rm eff})_{\rm isothermal}$, we solve Eq. (2) for $L_a\bar{\tau}_a$ and divide the result by itself except with $\Delta\sigma$ set equal to zero, which gives

$$\frac{\dot{Q}L_{\rm eff}}{(\dot{Q}L_{\rm eff})_{\rm isothermal}} =$$
$$1 - \frac{(1+k)\Delta\sigma}{\sigma_L}\Big/\left\{\left(\frac{2t}{w} + \frac{\pi}{4} - 1\right) - \frac{\rho g}{\sigma_L}\left(wt - \frac{\pi w^2}{16}\right)\frac{L}{w}\sin\alpha\right\} \tag{5}$$

It remains to derive the expression for the net viscous force acting on the liquid by the groove walls in the gas-blocked region. For this we turn to Hufschmidt,[1] who carried out a study on the shearing effect of vapor flow on liquid in rectangular grooves. The results also apply to flow induced by a surface-tension gradient with $d\sigma/dz$ replacing the vapor shear τ_v on the groove surface. In the gas-blocked region the absence of condensation requires that there be no net flow of liquid across a given groove cross section. The surface flow toward the condenser end exactly equals the subsurface flow toward the evaporator end. This is a special condition in Hufschmidt's more general study, and for it he gives the ratio of the pressure gradient to the surface shear. For our formulation of the problem, we require the net shear force $f\,dz$ of the groove wall on an element of liquid of length dz. This force is related to pressure gradient dP/dz and the surface shear τ_v by a force balance on the element. The result can be written as

$$\frac{f}{\tau_v w} = \left[\left(-\frac{dP}{dz}\right)\frac{t}{\tau_v}\right] - 1 = k \tag{6}$$

where the reciprocal of the term in brackets is given by Hufschmidt as a function of $2t/w$. From his result we calculate the constant k as a function of t/w, which is shown in Fig. 3. The total net force of the groove walls on the liquid in the gas-blocked region is obtained by integrating f from Eq. (6) over the gas-blocked length. With τ_v given by $d\sigma/dz$, the result is $kw\Delta\sigma$, which was stated previously in Fig. 2.

As a numerical example typical of the degradation predicted by Eq. (5), we use the parameters for a heat pipe we tested.[2] It was fabricated from the aluminum extrusion developed for the ATS-6 satellite. The length was 124 cm. The heat pipe had a wick-lined gas reservoir, with a reservoir-to-condenser volume ratio of 10. The working fluid was ammonia. The grooves had a width w of 0.061 cm and a depth t of 0.109 cm, which gives 1.79 for the ratio t/w. Other parameters of the test condition were an evaporator elevation of 0.38 cm, which gives $\sin\alpha = 0.0031$, an active-section temperature of 21°C, which gives $\rho g/\sigma_L = 28.2$ cm^{-2}, and a condenser-end temperature of −51°C, which gives $\Delta\sigma/\sigma_L = 0.86$. For this condition, Eq. (5) gives as the capacity degradation $\dot{Q}L_{\rm eff}/(\dot{Q}L_{\rm eff})_{\rm isothermal} = 0.56$.

The measured capacity under isothermal conditions before the control gas was added to the heat pipe was 84 W-m. As the theory predicts, this capacity was unchanged after the gas was added and the sink temperature was raised sufficiently that all the gas was contained in the reservoir and the condenser was isothermal ($\Delta\sigma/\sigma_L = 0$). As the sink temperature was lowered, gas blockage of the condenser began and the capacity degradation set in. The maximum degradation occurred at the lowest sink temperature that could be achieved in the test apparatus, which gave a condenser-end temperature of −51°C and a nearly completely gas-blocked condenser. The measured capacity, with the reduced effective length taken into account, was 29 W-m, and therefore the capacity degradation was $\dot{Q}L_{\mathrm{eff}}/(\dot{Q}L_{\mathrm{eff}})_{\mathrm{isothermal}}=0.35$.

Conclusions

Clearly, the simple theory presented shows that the Marangoni effect can produce the large capacity degradation observed in gas-loaded axially grooved heat pipes, and correctly predicts how the degradation increases with the surface-tension difference across the condenser.

The difference between the measured degradation (65%) and that predicted (44%) can be largely attributed to idealizations used in the theoretical model, such as a flat meniscus in the condenser, rectangular rather than rounded groove lands, neglect of liquid density variation with temperature, and a zero liquid contact angle. For example, a contact angle of 30 deg would alone increase the predicted degradation from 44% to 56%.

Acknowledgments

The authors gratefully acknowledge the contribution to this work by T. Cochran and P. Simons of NASA Lewis Research Center, who first called our attention to the possible influence of Marangoni flow on variable-conductance heat-pipe hydrodynamics.

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Readers' Forum

Brief discussions of previous investigations in the aerospace sciences and technical comments on papers published in the AIAA Journal *are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one* Journal *page including formulas and figures. A discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.*

Velocity Coupling in Oscillatory Combustion of Solid Propellants

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THE phenomenon of oscillatory combustion instability in solid rocket motors results from the responsiveness of the combustion process to oscillations in the flow environment. Early studies have concentrated on response to pressure oscillations,[1-5] described as perpendicularly incident waves with alternate compression and expansion of the combustion zone. Analytical descriptions are one-dimensional models, which assume the propellant to be effectively homogeneous and isotropic. Such a model yields a "response function" relating the amplitude and phase of the "burning rate" oscillations to the pressure oscillations, which is dependent on oscillation frequency, propellant burning rate, and properties of the propellant. The combustion environment (mean or time-average properties) enters in primarily through the effect of mean pressure on mean burning rate, which "disappears" into a normalized frequency $\Omega=\alpha f/\bar{r}^2$ where f is frequency, α is thermal diffusivity of the propellant, and r is mean burning rate of the propellant. The central point to note here for the present objective is that the "pressure coupled response function" becomes viewed as a property of the propellant, not of location on the burning surface of a propellant charge. The weak dependence on pressure and limited change in mean pressure with location on the burning surface of the propellant charge are usually neglected in calculations of overall combustor stability, reinforcing the idea that the combustion response is a property of the propellant, rather than of location in the flowfield, on the surface of the propellant charge, or in the acoustic field.†

Now it has always been recognized that the nature of the gas flowfield adjoining the burning surface can affect mean burning rate,[6,7] and the effect (called "erosive burning") has been linked (by experiments and analytical models) to the mainstream flow velocity or mass flux. The erosive burning effect presumably depends on such detailed behavior as enhanced heat transfer, mixing in the combustion zone, and shearing stress at the propellant surface. Describing these effects analytically is a very formidable problem. However, for the present purpose it is sufficient to note that the erosive burning effect is a function not only of the propellant but also of some minimum set of flow variables descriptive of the flowfield near the burning surface (and encompassing the gas phase reaction layer). In a given combustor, this will correspond to a spatially nonuniform effect, which property is completely inescapable for erosive burning. As noted earlier, efforts have been made to simplify this complex problem by describing the flow dependence in terms of variables descriptive of the "mainstream" flow parallel to the burning surface, variables that are average values of the variable over a cross section of the flow channel. In other words, the combustor flow parallel to the burning surface is

Received April 12, 1979.

Index categories: Combustion Stability, Ignition, and Detonation; Solid and Hybrid Rocket Engines.

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†Of course, the effect of the combustion response on the pressure waves is a function of position on the acoustic mode.